# Problem –11

*Due in DSC 235 by 12 noon, Friday,* **December 1**, *2017*

**Problem A:** *Let* $(a_n)_{n=1}^{\infty}$ *be a sequence of positive real numbers with all terms different from 1. Show that if* $\lim_{n\to\infty} a_n = 1$, *then*

$$\lim_{n\to\infty} \frac{\ln(a_n)}{a_n - 1} = 1.$$

**Problem B:** *Let* $(a_n)_{n=1}^{\infty}$, $(b_n)_{n=1}^{\infty}$ *be sequences of positive real numbers such that*

$$\lim_{n\to\infty} a_n = a > 0 \quad \textit{and} \quad \lim_{n\to\infty} b_n = b > 0.$$

*Suppose that* $p, q > 0$ *satisfy* $p + q = 1$. *Prove that*

$$\lim_{n\to\infty} (pa_n + qb_n)^n = a^p b^q$$

**Problem C:** *Find the limit of the sequence* $(a_n)_{n=1}^{\infty}$, *where*

$$a_n = \left(1 + \frac{1}{n^2}\right) \cdot \left(1 + \frac{2}{n^2}\right) \cdot \ldots \cdot \left(1 + \frac{n}{n^2}\right), \qquad \textit{for } n = 1, 2, 3, \ldots$$

RULES:

- The competition is open to all *undergraduate* UNO students.
- Please submit your solutions to Andrzej Roslanowski in DSC 235 or to his mailbox. (Needless to say, they should be be written clearly and legibly.)
- The winners will be determined each semester based on the number of correct solutions submitted.
- Due to the Thanksgiving Day holidays, **solutions to POW-11 are due on 12/01/17**, two weeks after problems are posted. Happy Holidays!

PRIZES:

- Winners will receive books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.